

Chapter 2

Boolean Algebra and Logic Gates

The deadline for submitting answers is in the fifth week

1. Demonstrate the validity of the following identities by means of truth tables:
 - (a) DeMorgan's theorem for three variables: $(x + y + z)' = x'y'z'$ and $(xyz)' = x' + y' + z'$
 - (b) The distributive law: $x + yz = (x + y)(x + z)$
 - (c) The distributive law: $x(y + z) = xy + xz$
 - (d) The associative law: $x + (y + z) = (x + y) + z$
 - (e) The associative law and $x(yz) = (xy)z$
2. Simplify the following Boolean expressions to a minimum number of literals:
 - (a) $xy + xy'$
 - (b) $(x + y)(x + y')$
 - (c) $xyz + x'y + xyz'$
 - (d) $(A + B)(A' + B)'$
 - (e) $(a + b + c')(a'b' + c)$

- (f) $a'bc + abc' + abc + a'bc'$

3. Simplify the following Boolean expressions to a minimum number of literals:

- (a) $ABC + A'B + ABC'$
- (b) $(x + y)'(x' + y')$
- (c) $(BC' + A'D)(AB' + CD')$
- (d) $x'yz + xz$
- (e) $xy + x(wz + wz')$

4. Reduce the following Boolean expressions to the indicated number of literals:

- (a) $A'C' + ABC + AC'$ to three literals
- (b) $(x'y' + z)' + z + xy + wz$ to three literals
- (c) $A'B(D' + C'D) + B(A + A'CD)$ to one literal
- (d) $(A' + C)(A' + C')(A + B + C'D)$ to four literals
- (e) $ABC'D + A'BD + ABCD$ to two literals

5. Find the complement of $F = wx + yz$; then show that $FF' = 0$ and $F + F' = 1$. Find the

6. complement of the following expressions:

- (a) $xy' + x'y$
- (b) $z + z'(v'w + xy)$
- (c) $(a + c)(a + b')(a' + b + c')$

7. Given the Boolean functions F_1 and F_2 , show that

- (a) The Boolean function $E = F_1 + F_2$ contains the sum of the minterms of F_1 and F_2 .
- (b) The Boolean function $G = F_1F_2$ contains only the minterms that are common to F_1 and F_2 .

8. List the truth table of the function:

- (a) $F = xy + xy' + y'z$
- (b) $F = bc + a'c'$

9. Draw logic diagrams to implement the following Boolean expressions:

- (a) $y = [(u + x')(y' + z)]$
- (b) $y = (u \oplus y)' + x$
- (c) $y = (u' + x')(y + z')$
- (d) $y = u(x \oplus z) + y'$
- (e) $y = u + yz + uxy$
- (f) $y = u + x + x'(u + y')$

10. Implement the Boolean function

$$F = xy + x'y' + y'z$$

- (a) With AND, OR, and inverter gates
- (b) With OR and inverter gates
- (c) With AND and inverter gates
- (d) With NAND and inverter gates
- (e) With NOR and inverter gates

11. Simplify the following Boolean functions T_1 and T_2 to a minimum number of literals:

A	B	C	T_1	T_2
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

12. The logical sum of all minterms of a Boolean function of n variables is 1.

- (a) Prove the previous statement for $n = 3$.
- (b) Suggest a procedure for a general proof.

13. Obtain the truth table of the following functions, and express each function in sum-of-minterms and product-of-maxterms form:

- (a) $(b + cd)(c + bd)$
- (b) $(cd + b'c + bd')(b + d)$
- (c) $bd' + acd' + ab'c + a'c'$
- (d) $(c' + d)(b + c')$

14. For the Boolean function

$$F = xy'z + x'y'z + w'xy + wx'y + wxy$$

- (a) Obtain the truth table of F .

- (b) Draw the logic diagram, using the original Boolean expression.
- (c) Use Boolean algebra to simplify the function to a minimum number of literals.
- (d) Obtain the truth table of the function from the simplified expression and show that it is the same as the one in part (a).
- (e) Draw the logic diagram from the simplified expression, and compare the total number of gates with the diagram of part (b).

15. Express the following function as a sum of minterms and as a product of maxterms:

$$F(A, B, C, D) = B'D + A'D + BD$$

16. Express the complement of the following functions in sum-of-minterms form:

- (a) $F(A, B, C, D) = \sum(2, 4, 7, 10, 12, 14)$
- (b) $F(x, y, z) = \prod(3, 5, 7)$

17. Convert each of the following to the other canonical form:

- (a) $F(x, y, z) = \sum(1, 3, 5)$
- (b) $F(A, B, C, D) = \prod(3, 5, 8, 11)$

18. Convert each of the following expressions into sum of products and product of sums:

- (a) $(u + xw)(x + u'v)$
- (b) $x' + x(x + y')(y + z')$

19. Draw the logic diagram corresponding to the following Boolean expressions without simplifying them:

- (a) $BC' + AB + ACD$
- (b) $(A + B)(C + D)(A' + B + D)$
- (c) $(AB + A'B')(CD' + C'D)$
- (d) $A + CD + (A + D')(C' + D)$

20. Write Boolean expressions and construct the truth tables describing the outputs of the circuits described by the logic diagrams in the following table

f_1	f_2	a	b	c
1	1	0	0	0
0	1	0	0	1
1	0	0	1	0
1	1	0	1	1
1	0	1	0	0
0	1	1	0	1
1	0	1	1	1

21. Write the following Boolean expressions in sum of products form:

$$(b + d)(a' + b' + c)$$

22. Write the following Boolean expression in product of sums form:

$$a'b + a'c' + abc$$